Parameter optimization for tandem regenerative system based on critical path*

LI Yongjian (李勇建)** and TU Fengsheng (涂菶生)

Department of Automation, College of Information Science and Technology, Nankai University, Tianjin 300071, China

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Abstract For a tandem queue system, the regenerative path is constructed. In an inter-regeneration cycle, the sensitivity value of performance measure with respect to the adjustable parameter θ can be acquired based on a fixed length of observation. Furthermore, a new algorithm of parameter optimization for the tandem queue system is given, which requires less simulation and no analysis for the perturbation transmission and makes a better estimation for the sensitivity.

Keywords: discrete event dynamic system, tandem regenerative system, perturbation analysis, critical path, stochastic optimization.

Many complex stochastic systems can be modeled as discrete event dynamic system (DEDS), such as communication network, computer system, production line, and program evaluation and review technique(PERT)-project network. Due to complicated interactions of such discrete events over time, it is important to study their performance and the optimization of their parameters. The tandem queue system has been the object of many studies in DEDS. Many researches were focused on deducing the analytical solutions or empirical formulas for performance measure[1]. Typical analytical tool is the queue theory, by which many useful conclusions in modeling, controlling and optimization for DEDS have been reached. However, analytical solutions cannot be acquired easily for many DEDS, so the discrete event simulatinon that emerged in recent years provides an additional set of design tools and performance evaluation techniques. However, the simulation is a computationally expensive and time-consuming process. And, the proper design of a simulation experiment for a complex stochastic system is itself a difficult task.

In recent years, several new approaches have been developed aiming at improving the efficiency of simulation, namely extracting as much information as possible from a single simulation run. A technique, known as perturbation analysis (PA) methodology[2,3], was originally developed to obtain from a single sample path the sensitivity estimates $\frac{\mathrm{d}J(\theta)}{\mathrm{d}\theta}$ of performance measures $J(\theta)$ of interest with respect to parameter vector θ. However, it is difficult to analyze the generation and transmis-

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** E-mail: yj-li@263.net.

sion of perturbation in the sample path and the choice of $\Delta\theta_i$ when we adopt PA method, which, in most cases, is a tedious task. So the critical path is proposed to be the nominal sample path. The concept of the critical path[3,5] simplifies PA method by reducing the problem of PA to one analyzing the event of the critical path. Therefore, the parameter optimization of DEDS becomes not only intuitive, but also effective. By combining PA method with the stochastic approximation, an algorithm for optimizing the system parameter (perturbation-analysis-robbins-monro-single-run, PARMSR) can be obtained[3], which, based on observation of a fixed length (e.g. L items), can be used to compute optimal parameter θ^* to optimize the performance function $J(\theta)$.

The PARMSR algorithm has a preferable convergent rate[3,6]. However, there are two problems for the algorithm that should be solved. One is how to determine the length L of the simulated sample path, and the other is how to estimate $\frac{\mathrm{d}J(\theta)}{\mathrm{d}\theta}$ effectively using PA method. The introduction of the critical path makes it effective to compute $\frac{\mathrm{d}J(\theta)}{\mathrm{d}\theta}$ using PA method. However, when L becomes very large, the number of critical paths may become enormous in some cases; as a result it is difficult to compute the sensitivity based on the critical path. Therefore, we should estimate the value L too.

In the paper, the regenerative tandem queue system with finite buffers (for short, the tandem regenerative system) is considered. Firstly, the model for the system is described, and the regenerative process for tandem queue system is constructed. Then, the critical path is proposed and its steady properties are discussed. Finally, the parameter optimization for the tandem regenerative system is considered, and an optimization algorithm is proposed.

1 Construction of regenerative process for tandem queue system

Many problems in DEDS can be regarded as the tandem queue system as shown in Fig. 1. The system consists of S stations labeled $M_1, M_2, \cdots, M_S$, in which there

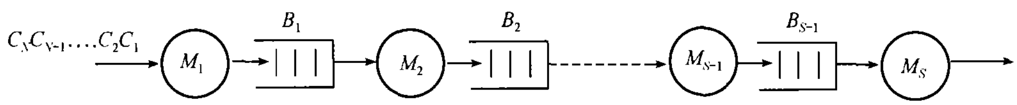

Fig. 1 Tandem queue system with finite buffers.

may be buffer spaces B_i between stations M_i and M_{i+1} to diminish the occurrence of blocking. We denote by b_i the capacity of the buffer B_i, $i = 1, 2, \cdots, S-1$. Suppose $P_i[k, \theta]$ is the time period during which item C_k, the kth item, enters the system and is processed in station M_i, where θ is the control parameter vector, $1 \leqslant i \leqslant S, k \geqslant 1$. For simplicity, we write $P_i[k]$ instead of $P_i[k, \theta]$. Suppose the time of the item C_k arriving at the station M_1 is T_k, the arrival interval between item C_k and C_{k+1} is A_k, $A_k = T_{k+1} - T_k$, and $\{A_k, k \geqslant 1\}$ is an i. i. d. (independent identical distribution) stochastic sequence, with the distribution $F_a(x)$ and expectation $\mathrm{E}A_k \triangleq \frac{1}{\lambda}$. When i is fixed, $\{P_i[k], k \geqslant 1\}$ is an i. i. d. stochastic sequence with the distribution $G^{(i)}(x, \theta)$ and density function $g^{(i)}(x, \theta)$, $u_i(\theta) = \{G^{(i)}(x, \theta)\}^{-1}$, $\mathrm{E}P_i[k] = \frac{1}{u_i(\theta)} < \infty$. When i is variable, $\{P_i[k],$

$k \geqslant 1\}$ are independent of each other, and independent of $\{A_k, k \geqslant 1\}$. Suppose that the system is empty when the item C_1 arrives at the first station M_1. We give the definition of the regeneration as follows.

Definition 1. A stochastic process $\{X_n, n \geqslant 1\}$ is a regenerative process if there exists a renewal process $\tau_k = N_1 + \cdots + N_k, k \in N^+$, $N+ \triangleq \{1,2,\cdots\}$ ($\{N_k\}$ is i.i.d. and finite a.s.) such that for each k, the process $\{X_{\tau_k + n}, n \in N^+\}$ is independent of $\{X_n, n \leqslant \tau_k\}$ and its distribution is independent of the index k. We refer to τ_k as the regeneration point, and N_k the kth inter-regeneration time cycle.

We denote by $X_i(n, \theta)$ the time of the item C_n departing from the station M_i, $1 \leqslant i \leqslant S$, $\forall n \geqslant 1$, and let $X_n^0 = \sum_{i=1}^{n} A_i$, $D_n(\theta) = X_S(n, \theta) - X_n^0$. Then $D_n(\theta)$ is the time period during which item C_n passes through the system. Ref. [7] indicated that, if $\forall 1 \leqslant i \leqslant S$, $\lambda \leqslant u_i(\theta)$, then $\{D_n(\theta), n \geqslant 1\}$ reaches a steady state after finite steps, and $D(\theta) = \lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} D_i(\theta)$ exists.

If the system empties infinitely often a.s., then the system is regenerative. Ref. [8] gave the sufficient and necessary condition for the system to empty infinitely often a.s. But this is not the case for most systems. Therefore, we need to reconstruct the sequence of regeneration points, so that the sequence can be applied to a more general tandem system.

Let $\boldsymbol{P}_n = (P_1[n], P_2[n], \cdots, P_S[n])$, $\boldsymbol{W}_n = (W_1[n], W_2[n], \cdots, W_S[n])$ and $\boldsymbol{Y}_n = (T_n - T_{n-1}, \boldsymbol{P}_n, \boldsymbol{W}_n)$, where $W_i[n] (1 \leqslant i \leqslant S)$ is the time period during which item C_n is waiting in the station M_i.

We define $\{\sigma^{(l)}\}$ as a stochastic sequence satisfiying

$$\sigma^{(l)} = \begin{cases} 0, & l = 0, \\ \inf\{l > \sigma^{(l-1)} : \boldsymbol{W}_l = 0\}, & l = 1,2,\cdots. \end{cases}$$

Obviously, when the number of stations $S \geqslant 2$, the process $\{\sigma^{(l)}\}$ is not a renewal process; therefore it cannot be used as the sequence of regeneration points. Ref. [8] proved that a sub-sequence of $\{\sigma^{(l)}\}$ is a renewal process; therefore, it can constitute a sequence of regeneration points.

For arbitrary $\boldsymbol{a} = (a_1, a_2, \cdots, a_S) = \overbrace{(0, +\infty) \times (0, +\infty) \times \cdots \times (0, +\infty)}^{S}$, letting $\boldsymbol{a}' = (a_1, a_2, \cdots, a_{S-1})$ and $\boldsymbol{a}^* = (a_1, a_1 + a_2, \cdots, a_1 + \cdots + a_S)$, we define stochastic time $\{\tau_a^{(l)}\}$ as

$$\tau_a^{(l)} = \begin{cases} 0, & l = 0, \\ \inf\{n > \tau_a^{(l-1)} : (\boldsymbol{W}_{n-1} + \boldsymbol{P}_{n-1})^* \leqslant \boldsymbol{a}^*, (T_n - T_{n-1}, \boldsymbol{P}_n) \geqslant \boldsymbol{a}\}, & l = 1,2,\cdots. \end{cases} \tag{1}$$

Obviously, for any l, $\tau_a^{(l)}$ is a regeneration point of Markov chain $\{\boldsymbol{Y}_n\}$. And for $n = \tau_a^{(l)}$, we have

$$(\boldsymbol{W}_{n-1} + \boldsymbol{P}_{n-1})^* \leqslant (T_n - T_{n-1}, \boldsymbol{P}'_n)^*. \tag{2}$$

In this case, for any $\tau_a^{(l)} < \infty$, we have

$$\boldsymbol{W}_{\tau_a^{(l)}} = 0. \tag{3}$$

Lemma 1[8]. Let $\boldsymbol{a} \in R_+^S$ such that $\gamma_a \triangleq F(a^{(1)}, \infty) G^{(1)}(a^{(2)}, \infty) \cdots G^{(S-1)}(a^{(S)}, \infty) < 0$ and $\boldsymbol{a}^* > \boldsymbol{g}^*$. Then $(\tau_a^{(l)}, l = 1, 2, \cdots)$ is a renewal process, the Markov chain $(\boldsymbol{Y}_n;\ n = 0, 1, \cdots)$ is regenerative with respect to $(\tau_a^{(l)})$, and the random variable $\boldsymbol{Y}_{\tau_a^{(l)}}$ obeys the distribution

$$\boldsymbol{\nu}_a = F_{a^{(1)}} \otimes G^{(1)}_{a^{(2)}} \otimes \cdots \otimes G^{(S-1)}_{a^{(S)}} \otimes G^{(S)} \otimes \varepsilon_0. \tag{4}$$

For the tandem queue system with finite buffers, to make the system avoid being blocked when buffer B_{i-1} is full, the following condition should be met:

$$\frac{T_n - T_{n-1}}{g^{(i)}} + b_{i-1} \geqslant_{st} \frac{T_n - T_{n-1}}{g^{(i-1)}}, \tag{5}$$

where $\geqslant_{st}$ is a stochastic order. Because $T_n - T_{n-1}$ and $T_1 - T_0 = T_1$ are i.i.d., the formula can be expressed as $[g^{(0)}, \boldsymbol{g}'] \geqslant_{st} \boldsymbol{g} + T_1[\bar{b}^{(0)}, \bar{b}]$, where $\bar{b} = (b_1^{-1}, b_2^{-1}, \cdots, b_{S-1}^{-1})$, $\bar{b}^{(0)}$ is an arbitrarily given number and $g^{(0)}$ is a stochastic variable with $g^{(0)} \geqslant_{st} g(1) + T_1 \bar{b}^{(0)}$.

Formula (5) ensures that in a regeneration cycle, the sum of the number of items being processed in station M_i and the capacity b_{i-1} of buffer B_{i-1} is stochastically larger than the number of items being processed in station M_{i-1}.

Therefore, for the tandem queue system with finite buffers, we construct the regenerative process based on the theorem as follows.

Theorem 1. Let $\boldsymbol{a} \in R_+^S$. $\boldsymbol{\nu}_a$ is prescribed in Formula (4), with $\boldsymbol{a}^* > \boldsymbol{g}^*$ and $[g^{(0)}, \boldsymbol{g}'] \geqslant_{st} \boldsymbol{g} + T_1[\bar{b}^{(0)}, \bar{b}]$. Then $(\tau_a^{(l)}, l = 1, 2, \cdots)$ is a renewal process. Let $n = \tau_a^{(l)}$. Then $\boldsymbol{X}_n = (X_1[n], X_2[n], \cdots, X_S[n])$ is a regenerative process relative to $\tau_a^{(l)}$.

Proof. According to the characteristics of the tandem queue system, $X_i[k]$, the time of item C_k departing from station M_i, satisfies

$$\begin{cases} X_1[k] = X_1[k-1] + P_1[k], \\ X_i[k] = \max(X_{i-1}[k], X_i[k-1]) + P_i[k]. \end{cases} \tag{6}$$

Firstly, from (1), $\forall\, l \geqslant 0$, $\tau_a^{(l+1)} - \tau_a^{(l)}$ depends only on the waiting time and processing time of the item C_k with $k \geqslant \tau_a^{(l)}$; therefore, $(\tau_a^{(l)}, l = 1, 2, \cdots)$ is a renewal process, which can be a sequence of regeneration points.

When $k = \tau_a^{(l)}$, by (3), $\boldsymbol{W}_{\tau_a^{(l)}} = 0$, so $X_i[k] = P_i[k] + X_{i-1}[k]$.

When $k > \tau_a^{(l)}$, $X_i[k] = \max(X_i[k], X_i[k-1]) + P_i[k]$, which depends only on the time

$X_i[k]$ for $k \geqslant \tau_a^{(l)}$, and is not related to $X_i[k]$ for $k < \tau_a^{(l)}$. Because $\boldsymbol{W}_{\tau_a^{(l)}} = 0$, we can regard $C_{\tau_a^{(l)}}$ as C_1, which is not related to l. Therefore, according to Definition 1, $\{X_n\}$ is regenerative with respect to $\tau_a^{(l)}$.

Remark 1. When the traffic intensity is smaller than 1, that is, $\int tF(\mathrm{d}t) > \max \int tG_i(\mathrm{d}t)$, there must exist $\boldsymbol{a}$ satisfying (4) and $\boldsymbol{a}^* > \boldsymbol{g}^*$. For example, we can choose $\varepsilon > 0$ to be so small that $f - \varepsilon > \max g(i)$, and set $\boldsymbol{a} = (f - \varepsilon, \boldsymbol{g}')$.

Remark 2. $W_1[n-1] + P_1[n-1] = X_1[n-1]$, $W_1[n-1] + P_1[n-1] + W_2[n-1] + P_2[n-1] = X_2[n-1]$. Furthermore, for $i = 1, 2, \cdots, S$, $W_1[n-1] + P_1[n-1] + \cdots + W_i[n-1] + P_i[n-1] = X_i[n-1]$. Since $(\boldsymbol{W}_{n-1} + \boldsymbol{P}_{n-1})^* = (W_1[n-1] + P_1[n-1], W_1[n-1] + P_1[n-1] + W_2[n-1] + P_2[n-1], \cdots)$, Formula (1) can be expressed as

$$\tau_a^{(1)} = \inf\{n > \tau_a^{(l-1)} : X[n-1] \leqslant \boldsymbol{a}^*, (T_n - T_{n-1}, \boldsymbol{P}'_n) \geqslant \boldsymbol{a}\}. \tag{7}$$

2 Critical path and its stability of tandem queue system

With the tandem queue system in Fig. 1, the processing of an item on a station is called an event. We consider the connection between any two events. Denote the operation on item C_k in station M_i by $O_{i,k}$. Let $S(O_{i,k})$, $F(O_{i,k})$ and $m(O_{i,k})$ be the beginning time, finishing time and time period of event $O_{i,k}$, respectively. If $X_i[k] > F(O_{i,k})$, we say that event $O_{i,k}$ is blocked. In this case, the almost simultaneous event[5] of the event $O_{i,k}$ is

$$O_{i+1,k-b_i}, O_{i+2,k-b_i-b_{i+1}}, \cdots, O_{i+l(i,k),k-b_i-\cdots-b_{i+l(i,k)-1}},$$

where $l(i,k) = \min\{S - i, \arg\min\{j \mid k \leqslant b_i + \cdots + b_{i+j-1}\}\}$. We call event $O_{i-1,k}$ and its almost simultaneous event the almost tight event of event $O_{i,k}$. If $O_{i',k'}$ is an almost tight event of event $O_{i,k}$ and $S(O_{i,k}) = F(O_{i',k'}) = X_{i'}[k']$, then event $O_{i',k'}$ is a tight event of event $O_{i,k}$, expressed as $O_{i',k'} < O_{i,k}$. Obviously, if $O_{i'',k''} < O_{i',k'}$ and $O_{i',k'} < O_{i,k}$, then $O_{i'',k''} < O_{i,k}$. Therefore, "$<$" is a partial order in the event set.

For arbitrary two events $O_{i',k'}$ and $O_{i,k}$, if there are events $O_1, O_2, \cdots, O_l$ such that $O_1 = O_{i',k'}$, $O_l = O_{i,k}$ and $O_{r-1} < O_r (r = 2, \cdots, l)$, then we call $w(i', k'; i, k) = O_1 O_2 \cdots O_\tau$ a path from event $O_{i',k'}$ to $O_{i,k}$. If event O_{r-1} is a tight event of O_r, then we call $w(i', k'; i, k)$ a critical path from event $O_{i',k'}$ to $O_{i,k}$, denoted by $w^*(i', k'; i, k)$. Let $m(w)$ be the total processing time of all events in the path w. Then $m(w^*) = \sum_{r=1}^{l} m(O_r)$. Let $C(i', k'; i, k)$ and $A(i', k'; i, k)$ be the collection of critical paths and paths from event $O_{i',k'}$ to $O_{i,k}$, respectively. Then we have a lomma as follows.

Lemma 2[4]**.** If $w^*(l', k'; i, k) \in C(i', k', i, k)$, then

$$m(w^*(i', k'; i, k)) = \max\{m(w) \mid w \in A(i', k'; i, k)\}.$$

Choose the mean processing time of an item through the tandem queue system as the performance objective. Then $J(\theta) = \lim_{N\to\infty} \frac{X_S[N,\theta]}{N}$. In Ref. [5], we have proved that $X_S[N,\theta] = m(w^*(1,1;S,N))$; therefore, $J(\theta) = \lim_{N\to\infty}\frac{1}{N}m(w^*(1,1;S,N))$.

When N is large, the cost of computing $w^*(1,1;S,N)$ is huge. To avoid it, we adopt the aforementioned regenerative process.

By Theorem 1, there must exist an infinite increasing sequence $\tau_a^{(k)} \to \infty$, $(k \to \infty)$. Then let $n = \tau_a^{(k)}$. $\{X_n\}$ is a regenerative process, which makes C_n non-blocked,

$$\begin{cases} X_1[n] = X_1[n-1] + P_1[n], \\ X_i[n] = X_{i-1}[n-1] + P_i[n-1], \qquad i = 2,\cdots,S. \end{cases}$$

Therefore, to acquire the steady value of $J(\theta)$, we just simulate $\tau_a^{(1)}$ items every time. According to the conclusions in Refs. [5,9], the following theorem can be proved.

Theorem 2. If $E\tau_a^{(1)} < \infty$, then $J(\theta) = \frac{1}{E\tau_a^{(1)}}E(m(w^*(1,1;S,\tau_a^{(1)})))$.

To improve the precision of estimating $J(\theta)$, we can simulate $\tau_a^{(p)}$ items. With the ith simulation, we can obtain $J_i(\theta) = \frac{1}{E\tau_a^{(i)}}Em(w^*(1,\tau_a^{(i-1)};S,\tau_a^{(i)})), i = 1,2,\cdots,p$, so we have $J(\theta) = \frac{1}{p}\sum_{i=1}^{p} J_i(\theta)$.

3 Parameter optimization of tandem regenerative system

For the tandem regenerative system, based on an algorithm of computing critical paths given by the principle of dynamic programming on Hasse graph in Ref. [10], an algorithm for parameter optimization of the tandem regenerative system is derived in this section.

Firstly, we should determine the number k of items to be simulated, $k = \tau_a^{(1)}$, where $\tau_a^{(1)}$ is the first regeneration point. From (7), we have

$$\tau_a^{(1)} = \inf\{n > 0 : \boldsymbol{X}[n-1] \leqslant \boldsymbol{a}^*, (T_n - T_{n-1}, \boldsymbol{P}'_n) \geqslant \boldsymbol{a}\}. \tag{8}$$

To compute the number $\tau_a^{(1)}$, we should first determine the value $\boldsymbol{a}$. According to Remark 1, we can choose a small enough value $\varepsilon > 0$ such that $f - \varepsilon > \max g(i)$, and then we let $\boldsymbol{a} = (f - \varepsilon, \boldsymbol{g}')$.

Having acquired the critical path, a perturbation analysis can be processed using the method described in Ref. [4] to compute $\frac{dJ(\theta)}{d\theta}$. Then from $\theta_{n+1} = \theta_n + c_n\frac{dJ(\theta)}{d\theta}$, we can estimate the optimal value θ^* of $J(\theta)$.

With the known performance function $J(\theta)$, we choose the optimal coefficient c_n by a linear search in the direction $\frac{\mathrm{d}J(\theta)}{\mathrm{d}\theta}$; that is, $c_n = \arg\min_c \left\{ J\left(\theta + c\frac{dJ(\theta)}{d\theta} \right) \right\}$. In our study, $w^*(1,1; S, \tau_a^{(k)})$ has no analytical expression, so only simulation can be adopted. The basic method is to choose m numbers $c_i = \frac{i}{m}c$, $i = 1,2,\cdots,m$ uniformly in interval $[0,c]$ (c is a constant). Then we get m points $\theta + c_i\frac{\mathrm{d}J(\theta)}{\mathrm{d}\theta}$, $i = 1,2,\cdots,m$, where a "good enough" point can be found among the m points with the ordinal optimization method[11]. So considering the corresponding c_i as the recursive coefficient, a recursion can be realized.

Based on the discussion above, an algorithm for the parameter optimization can be derived as follows.

Suppose the following parameters are known beforehand: density function f of items arriving randomly, density function vector $\boldsymbol{g}$ of service times in every station and the vector $\boldsymbol{b}$ of capacities in every buffer and error constant δ.

Step 1. Choose starting point $\boldsymbol{\theta}_0$, $k = 1$, $\delta > 0$, $\boldsymbol{a} = (f - \varepsilon, \boldsymbol{g}')$ with $\varepsilon < f - \max g(i)$, and $\tau_a^{(0)} = 0$.

Step 2. Compute $\tau_a^{(k)} = \inf\{n > 0: \boldsymbol{X}[n-1] \leqslant \boldsymbol{a}^*, (T_n - T_{n-1}, \boldsymbol{P}'_n) \geqslant 0\}$ to acquire the number $\tau_a^{(k)}$ of items to be simulated.

Step 3. Simulate $\tau_a^{(k)}$ items, and then compute the critical path $w^*(1, \tau_a^{(k-1)}, m, \tau_a^{(k)})$.

Step 4. Compute $\frac{\mathrm{d}J(\theta)}{\mathrm{d}\theta}$ using the method in Ref. [4].

Step 5. Choose m numbers $c_1, c_2, \cdots, c_m$ uniformly in the interval $[0,c]$, and then choose the recursive coefficient in the next step based on m vectors $\theta_k + c_i\frac{\mathrm{d}J(\theta)}{\mathrm{d}\theta}$, $(i = 1,2,\cdots,m)$ via the ordinal optimization.

Step 6. $\theta_{k+1} \leftarrow \theta_k + c_k\frac{\mathrm{d}J(\theta)}{\mathrm{d}\theta}$, $k \leftarrow k+1$.

Step 7. If $\| \theta_{k+1} - \theta_k \| \leqslant \delta$, then exit, else, go to Step 2.

4 Conclusions

In this paper, the regenerative properties of the tandem queue system with finite buffers have been discussed; the regenerative process has been constructed so that the computation of the steady value of the performance function can be reduced to the problem of simulating finite items. The result is applied to the parameter optimization, and the steady value of the performance function is obtained through finite simulations, so simulation of a large number of items is reduced simulation of a small quantity of items. The studies of regeneration, critical path and ordinal optimization have a common objective; that is, to reduce the number of simulations and at the same time to improve efficiency of simulation. Therefore, the combination of the three methods can certainly improve the efficiency of simulation. This is an interesting problem. We will discuss the complexity of the algorithm and improve the algorithm in a near future.

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